the approximate nature of the expression for $\rho(r)$, there is no point in precise study of the question of shock wave formation, if we assume it possible to smooth the function in the immediate vicinity of the point r_1 when necessary.

Analysis of Eq. (4) reveals that at $\alpha_2 > \alpha_1$ the efficiency of compression increases, i.e., $J(\alpha_1, \alpha_2) > J(\alpha_1)$, but compression of a solid target by the method based on the solution described above with self-similarity index $\delta = 1$ at $\alpha_2 > \alpha_1$ is impossible, since the constant c_1 must then exceed the value permitted by the energy specified. The latter can easily be demonstrated, since the solution in the vicinity of the origin is known and described by Eq. (2), and thus improvement of the technique described is impossible. In collapse of hollow target the constant c_1 can be increased for the same total applied energy since the energy can be expended almost totally in piston work up to the moment when the piston trajectory intersects the singular characteristic going toward the center. After the piston is halted, the rarefaction wave produces a density distribution in the peripheral portion of the target with $\alpha_2 > \alpha_1$, so that the efficiency of hollow target compression is increased.

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MATHEMATICAL MODELING AND CALCULATION OF EXPLOSION EFFECT IN CONTINUOUS MEDIA

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In the conversion of explosion energy into electrical or mechanical energy, the following problem arises. There exists a plasma sphere of radius r_*, characterized by parameters p_0, ρ_0, T_0. At time $t = 0$ there occurs an instantaneous expulsion of photons and high velocity microparticles from this sphere, and the sphere also begins to expand into a spherical cavity, the space outside which is occupied by a continuous medium with parameters p_1, ρ_1, T_1 (Fig. 1). The surrounding medium is assumed condensed and will be studied using a hydrodynamic description. We will also assume that the surrounding medium effectively absorbs the energy of particles formed during the explosion, so that the major portion of the explosion energy is transferred to the medium in some region about the center of energy liberation. To define the parameters of the motion which develops it is necessary to develop a mathematical model of the flow to be studied, i.e., to write equations of motion for the continuous medium interacting with the particles and light radiation, and to specify initial and boundary conditions.

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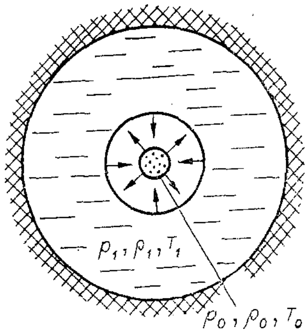

Fig. 1

The present study presents a mathematical model of these phenomena, and considers explosion in a cavity surrounded by water.

1. Equations of Motion. The complete system of equations describing behavior of the medium and the particle and photon flows propagating therein consists of the hydrodynamics equations which in this case must consider the contribution to forces acting on the medium and energy exchange with the medium of the particles and photons, and the transfer equations which consider scattering the absorption for particles and radiation.

We have the continuity equation

$$d\rho/dt + \rho \operatorname{div} \mathbf{v} = 0, \tag{1.1}$$

where ρ is density and v is the velocity of the medium. We assume the particle density to always be low, so that the particles produce no significant contribution to the medium's inertial properties.

The Navier—Stokes equation

$$\rho \frac{dv^i}{dt} = -\nabla^i p + \nabla_j \tau^{ij} + F^i_{(p)} + F^i_{(R)} + F^i_{(ex)}, \tag{1.2}$$

where $\nabla_j \tau^{ij}$ is the divergence of the viscous stress tensor

$$\tau^{ij} = \zeta_1 \left(\nabla^i v^j + \nabla^j v^i - \frac{2}{3} g^{ij} \nabla_k v^k\right) + \zeta_2 g^{ij} \nabla_k v^k, \tag{1.3}$$

where ζ_1 and ζ_2 are the viscosity coefficients, generally dependent on density and temperature of the medium. The forces appearing on the right side of Eq. (1.2), produced by the action on the medium of the particle flow $(F^i_{(p)})$ and the radiation $(F^i_{(R)})$, may be expressed in the form

$$F^i_{(p)} = -\frac{dp^i_{(p)}}{dt} - \nabla_j \tau^{ij}_{(p)}, \; F^i_{(R)} = -\frac{dp^i_{(R)}}{dt} - \nabla_j \tau^{ij}_{(R)}, \tag{1.4}$$

where $p^i_{(p)}, p^i_{(R)}$ are volume momentum densities; $\tau^{ij}_{(p)}$ and $\tau^{ij}_{(R)}$ are momentum flow densities of particles and radiation. The quantities $p_{(p)}, p^{(i)}_{(R)}, \tau^{ij}_{(p)}, \tau^{ij}_{(R)}$ are expressed in integral form in terms of the particle and photon distribution functions (corresponding to the functions presented below). The quantity $F^i_{(ex)}$ in Eq. (1.2) is the force external with respect to the system of medium + particles + radiation.

The heat increment equation

$$\frac{dU}{dt} = -p \frac{d}{dt}\left(\frac{1}{\rho}\right) + \frac{1}{\rho} \tau^{ij} e_{ij} + \frac{dq^{(e)}}{dt}, \tag{1.5}$$

where U is the internal energy.

The second term on the right (Gibbs formula) expresses uncompensated heat, while $e_{ij} = \nabla_i v_j + \nabla_j v_i$ is the medium deformation rate tensor. The third term on the right of Eq. (1.5) is the external addition of heat per unit volume of medium. This heat contribution, reduced to unit mass of the medium, is composed of the following components:

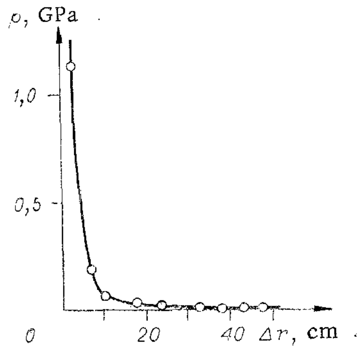

Fig. 2

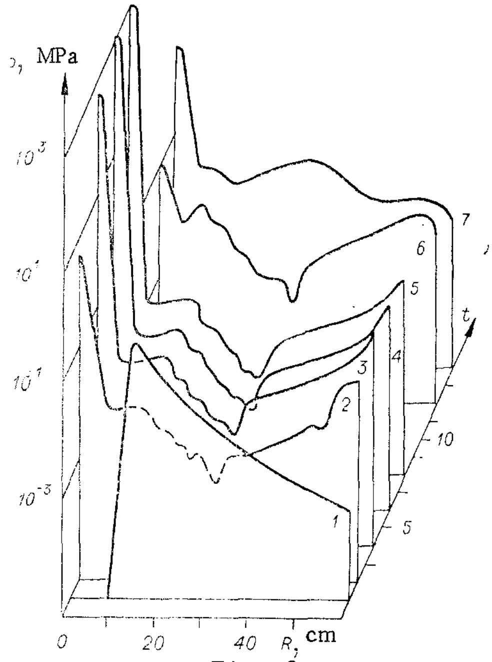

Fig. 3

$$\frac{dq^{(e)}}{dt} = \frac{1}{\rho}\nabla_j(\eta\nabla^j T) + \frac{dq^{(e)}_{(p)}}{dt} + \frac{dq^{(e)}_{(R)}}{dt} + \frac{dq^{(e)}_{(ex)}}{dt}. \tag{1.6}$$

Here the first term is produced by conventional thermal conductivity, while the thermal conductivity coefficient η may be dependent on density and temperature of the medium; the second and third terms are the heat added to a particle of the medium due to interaction with particle and radiation fluxes, respectively; the last term of Eq. (1.6) is the external heat added to the medium + particles + radiation system.

In analogy to Eq. (1.4), we represent $\rho dq^{(e)}_{(p)}/dt$ and $\rho dq^{(e)}_{(R)}/dt$ in the form

$$\rho\frac{dq^{(e)}_{(p)}}{dt} = -\frac{dE_{(p)}}{dt} - \nabla_k\Pi^k_{(p)},\ \rho\frac{dq^{(e)}_{(R)}}{dt} = -\frac{dE_{(R)}}{dt} - \nabla_k\Pi^k_{(R)}, \tag{1.7}$$

where $E_{(p)}$ and $E_{(R)}$ denote the volume energy densities of particles and photons, respectively, while $\Pi^k_{(p)}$ and $\Pi^k_{(R)}$ are the energy flux densities of particles and radiation.

The quantities $E_{(p)}$, $\Pi^k_{(p)}$, $\tau^{ij}_{(p)}$ for the particle flow, just like $E_{(R)}$ for the radiation, can be expressed in terms of particle N or radiation I_ν distribution functions. The function N is dependent on the spatial coordinates, the particle energy E, and the components of the unit vector ω^i. The quantity $N(x^k, E, \omega^i)dEd\Omega$ is the spatial density of number of particles in motion at a particular point with velocity vectors directed within the solid angle $d\Omega$ surrounding the vector ω^i and energies within the interval (E, E + dE). Similarly, the function $I_\nu(x^h, \nu, \omega^i)$ is defined so that the quantity $(1/c)I_\nu d\nu d\Omega$ is the spatial density of radiant energy propagating at a given point within the limits of the solid angle $d\Omega$ in the direction ω^i and lying in the frequency range (ν, $\nu + d\nu$). For the energy, momentum, and flux densities, we then have

$$\begin{aligned} &E_{(p)} = \int ENdEd\Omega,\ E_{(R)} = \frac{1}{c}\int I_\nu d\nu d\Omega, \\ &\Pi^i_{(p)} = \int \omega^i uENdEd\Omega,\ \Pi^i_{(R)} = \int \omega^i I_\nu d\nu d\Omega, \\ &p^i_{(p)} = \int \omega^i m_0 uNdEd\Omega,\ p^i_{(R)} = \frac{1}{c}\int \omega^i I_\nu d\nu d\Omega, \\ &\tau^{ij}_{(p)} = \int \omega^i\omega^j m_0 u^2 NdEd\Omega,\ \tau^{ij}_{(R)} = \int \omega^i\omega^j I_\nu d\nu d\Omega, \end{aligned} \tag{1.8}$$

where m_0 is the particle rest mass, and their velocity is $u = \sqrt{2E/m_0}$.

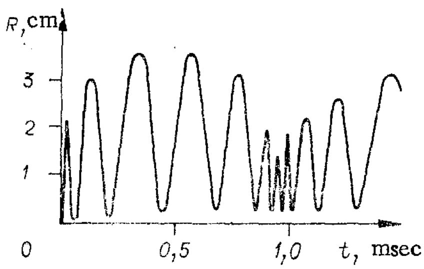

Fig. 4

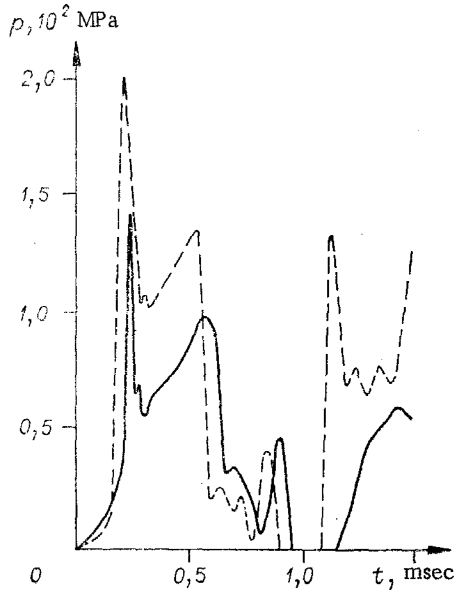

Fig. 5

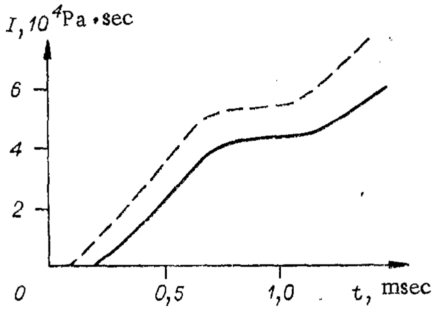

Fig. 6

The functions $N(x^k, E, \omega^i)$ and $I_\nu(x^k, \nu, \omega^i)$ must satisfy the following particle and radiation transfer equations.

The particle transfer equation [1]

$$\frac{1}{u}\frac{dN}{dt} + \omega^k \nabla_k N + \Sigma N = \int_0^\infty \Sigma_s(E') \int_\Omega g_{(p)}(E, E', \Omega, \Omega') N(E', \Omega') dE' d\Omega' + q_{(p)}. \tag{1.9}$$

Here $\Sigma = \Sigma_a + \Sigma_s$, Σ_a is the macroscopic absorption section; Σ_s is the particle scattering section; g(p) is the scattering function, the form of which is defined by additional hypotheses imposed upon elementary scattering and the properties of the particles participating.

The quantities Σ_a, Σ_s, $q_{(p)}$ appearing in Eq. (1.9) may depend on density, temperature, or other physicochemical parameters of the medium, and like the scattering function must be additionally defined either experimentally or commencing from some theoretical assumption.

Assuming localized thermodynamic equilibrium, the radiation transfer equation has the form

$$\frac{n_\nu}{c}\frac{d}{dt} I_\nu + \omega^k \nabla_k I_\nu + k_\nu I_\nu = k_\nu \Big[(1-\lambda_\nu) B_\nu(T) + \frac{\lambda_\nu}{4\pi} \int_0^\infty \int_\Omega I_\nu g_\nu(\nu, \nu', \omega, \omega', \mu_0) d\nu' d\Omega' \Big]. \tag{1.10}$$

Here n_ν is the index of refraction for the medium at frequency ν; $k_\nu = k_{a\nu} + k_{s\nu}$; $\lambda_\nu = k_{s\nu}/k_\nu$; $k_{a\nu}$ is the absorption coefficient; $k_{s\nu}$ is the scattering coefficient for radiation at frequency ν. Speaking generally, these coefficients may depend on temperature or density, and possibly other physicochemical characteristics of the medium, and are defined experimentally or theoretically. The function g_ν is the scattering function, defined by supplementary means; the function $B_\nu(T)$ has the form

$$B_\nu(T) = \frac{2\hbar\nu^3}{c^2}\frac{1}{(e^{\hbar\nu/kT}-1)}. \quad (1.11)$$

By using transfer equations (1.9)-(1.11) one can obtain expressions for the forces $F^i_{(p)}$ and $F^i_{(R)}$ in Eq. (1.2) and the heat increments $\frac{d}{dt}q^{(e)}_{(p)}$ and $\frac{d}{dt}q^{(e)}_{(R)}$ in Eq. (1.6) in terms of the particle and light quantum distribution functions. To do this it is sufficient to use Eqs. (1.4), (1.7), in which the right sides are calculated from the transfer equations by integrating the latter with a weight determined by Eq. (1.8). To close the system we must add to the above equations the equation of state of the medium, $T = F(U, \rho)$, $p = \Phi(U, \rho)$.

2. Approximate Analytical Description of the Effect of Microparticle Flux on the Absorbing Medium. We will consider the simplest possible model of interaction of the high-energy particle flux with the homogenous medium, initially at rest. We will assume further that the processes of particle absorption and scattering by the medium may be described approximately by some effective absorption coefficient. Moreover, we assume that the particles are absorbed by the medium and transfer energy to the latter in a time so brief that the medium does not become significantly displaced during the transfer. For this to be true it is obviously necessary that the mean time over which a particle is absorbed be much less than the characteristic hydrodynamic time of the problem. With this condition, the solution of the problem of behavior of the medium and particle and radiation fluxes propagating therein divides into two stages. The first stage is the solution of the transfer equations in the nonmoving medium with corresponding initial and boundary conditions, with subsequent determination of distributions of energy and momentum transferred to the medium by particles and energy due to absorption and scattering. Thus, the effect of particles and radiation on motion of the medium reduces to creation of a certain initial state, which must be taken as the initial conditions for solution of the thermodynamic problem. The second stage consists of solution of the hydrodynamic problem of motion of the medium with special (determined in the first stage) initial conditions.

We will consider the first stage of the problem, i.e., determining the state of the medium which is produced as a result of scattering and absorption by the medium of the high-energy particle fluxes. For definity we will assume that the particles exit from the center of a spherical cavity of radius r_0, outside which all space is filled with the absorbing medium.

As has already been indicated, we will assume that scattering and absorption can be described by the transfer equation with some effective absorption coefficient α, so that the transfer equation in the medium at rest will have the form

$$\frac{1}{v}\frac{dN}{dt} + \omega^k\nabla_k N + \alpha\Theta(r-r_0)N = \frac{N_0}{4\pi v}\delta(t)\delta(\mathbf{r}),$$

where $\Theta(r-r_0)$ is a Θ-function describing "inclusion" of absorption in the medium outside the cavity; N_0 is the total number of particles emitted into the medium and v is their velocity. On the right of this equation is a term describing a point impulsive particle source located at the origin.

For $r > r_0$ the solution of this equation has the form

$$N = \frac{N_0}{4\pi}\Theta(t)\frac{\delta(r-vt)}{r^2}e^{-\alpha(r-r_0)}\delta\left(\frac{\mathbf{r}}{r}-\omega\right). \quad (2.1)$$

Integration of Eq. (2.1) over angles provides an expression for the spatial particle density:

$$n(\mathbf{r}, t) = \int N d\Omega = \frac{N_0}{4\pi r^2}\Theta(t)\delta(r-vt)e^{-\alpha(r-r_0)}.$$

For the volume densities of energy and momentum transferred to the medium we have

$$\begin{aligned}\frac{dQ_{(p)}}{dt} &= \int E_0 N d\Omega = E_0 n(\mathbf{r}, t),\\ \frac{d\mathbf{p}_{(p)}}{dt} &= \int \omega N m_0 v d\Omega = m_0 v n(\mathbf{r}, t)\frac{\mathbf{r}}{r}.\end{aligned} \quad (2.2)$$

By integrating Eq. (2.2) over time, we obtain an expression for the total energy and momentum density transferred to the medium:

$$Q_{(p)} = \frac{\alpha \mathscr{E}}{4\pi r^2 \rho_1} e^{-\alpha(r-r_0)},$$
$$\mathbf{p}_{(p)} = \frac{\alpha m_0 \mathbf{v} N_0}{4\pi r^2 \rho_1} e^{-\alpha(r-r_0)} = \left(\frac{m_0 \mathbf{v}}{E_0}\right) Q_{(p)}. \qquad (2.3)$$

Here $\mathscr{E} = E_0 N_0$ is the total energy carried off by particles; ρ_1 is the density of the absorbing medium. In the problem of concern here, we may neglect the effect of $p^i_{(p)}$.

We will now calculate the energy liberated when the high-energy particle source is a highly compressed plasma sphere of radius ~10^{-2} cm and mass of ~10^{-3} g, expanding in a spherical cavity surrounded by water. During the period of sphere particle expulsion ~10^{-11} sec let there by expelled into the surrounding medium $N_0 \approx 10^{19}$ particles with mean energy $E_0 \approx$ 14.1 MeV per particle, carrying off 70% of the sphere's energy. The remainder of the sphere's initial energy is carried off by radiation or remains within the plasma as thermal energy of the expanding remains.

Substituting these quantities in Eq. (2.3) and taking the density of water ρ_1 = 1 g/cm^3 and the water absorption coefficient α = 0.05 cm^{-1}, we obtain

$$Q_{(p)} = 2\cdot 10^4 \left(\frac{1\text{cm}}{r}\right)^2 e^{-\alpha(r-r_0)} \left(\frac{\text{kal}}{\text{g}}\right). \qquad (2.4)$$

It is evident from Eq. (2.4) that at small cavity dimensions (r_0 ~ 1 cm) a significant amount of heat is liberated near the cavity boundary, while as cavity size increases the quantity of heat drops abruptly (at $r_0 \approx$ 40 cm $Q_{(p)} \approx$ 13 cal/g).

3. Solution of the Particle Transfer Equation. A simple approximation of the results of interaction between particles and the absorbing medium was given above. In view of the difficulties of analytically estimating errors in the medium's energy distribution, it is expedient to perform a direct calculation of the particle transfer by the Monte Carlo method. This technique allows solution of the complete integrodifferential transfer equation in which the scattering integrand is represented as the sum over all possible processes with secondary particle output. It will be assumed that energy transfer takes place instantaneously and directly at the collision point, and is not accompanied by any significant radiation. The goal of such a calculation will then be determination of the spatial distribution of the energy transferred by particles to the medium in the retardation process. The characteristics of the medium will be considered constant in solving the problem.

4. Solution of the Problem of Medium Motion. This problem may be solved by both analytical and numeric methods. If the medium is relatively incompressible (water) and the cavern radius is sufficiently large, then the perturbations will be weak and the flow of the medium may be studied analytically in the acoustic approximation. But if the cavity radius r_0 is small and a significant amount of energy is liberated into the medium then shock waves will develop and the problem must be solved numerically using the finite difference method.

In determining the initial parameters of the water only the total contribution of energy absorbed from the particle and radiation fluxes will be considered, and the quantities $\Pi^i_{(p)}, \Pi^i_{(R)}, F^i_{(p)}, F^i_{(R)}$, etc., [see Eq. (1.8)] will be neglected. The distribution of internal energy U within the water thus obtained is used together with the equation of state p = f(U, ρ) at a fixed density value to obtain the pressure distribution. The initial state of the nonmoving plasma sphere is considered homogeneous with a specified mass M = 10^{-3}g and density ρ_0 = 250 g/cm^3. The plasma itself is considered an ideal gas with specific heat ratio γ = $^5/_3$. To estimate the effect of radiation on the flow pattern, calculations were performed both with and without radiation. In all cases it was assumed that 30% of the initial energy remains within the plasma in the form of thermal energy.

The water was considered an ideal liquid with the following equations of state [2]:

thermal equations of state [2]

$$\text{at} \quad 1 \leqslant \rho \leqslant 2{,}3\ \text{g/cm}^3 \quad p = p_2(1 - 0{,}012\rho f) + 4{,}7\rho f(T - 273^\circ),$$

$$p_2 = [3050(\rho^{7,3} - 1)]/[1 + 0,7(\rho - 1)^4],$$

$$\begin{aligned} &\text{at} \quad \rho < 1\,\text{g/cm}^3 \; p = \xi^4 - 470\rho f + 4,7\,\rho f(T - 273^\circ), \\ &\xi = 6,6(1 - \rho)^{0,57}\rho^{0,25} \quad \text{at} \quad 0 \leqslant \rho \leqslant 0,8\,\text{g/cm}^3, \\ &\xi = 10(1 - \rho) + 66(1 - \rho^2) - 270(1 - \rho)^3 \quad \text{at} \quad 0,8 < \rho \leqslant 1, \\ &f = (1 + 3,5\rho - 2\rho^2 + 7,27\rho^6)/(1 + 1,09\rho^6); \end{aligned} \tag{4.1}$$

caloric equation of state

$$U = c_V T + \mathscr{F}(\rho), \tag{4.2}$$

where c_V is the heat capacity at constant volume, which is assumed constant. The function $\mathscr{F}(\rho)$ is defined such that both equations of state agree with each other.

In accordance with Eqs. (4.1), (4.2), the known distribution over coordinate r of the energy transferred to the water is used to calculate the initial pressure distribution. For comparison, Fig. 2 shows the initial pressure distributions p(r) in the water for $E_0 = 10^{14}$ erg and $r_0 = 1$ cm, obtained by Eq. (2.3) (solid line) and by the Monte Carlo method (points). The good agreement of the results is evident.

Below we will present results of calculation of the flow produced by liberation of an energy $E_0 = 10^{14}$ erg at several values of r_0 with a fixed thickness of the spherical water layer equal to 50 cm. It is assumed that the energy absorbed by the water comprises 70% of E_0. Moreover, it is assumed that the water mass is bounded externally by an absolutely rigid spherical wall.

The hydrodynamic calculations were performed by the finite difference method of [3] in Lagrangian coordinates. Within the framework of this method use of equations of state in the form of Eqs. (4.1), (4.2) avoids iteration procedures in determining the thermodynamic parameters of the water from the thermal increment equation.

The calculations revealed that the process develops in the following manner. Initially, the plasma cloud almost instantaneously radiates energy which is absorbed by the water, and the plasma temperature falls to 10^4 °K. During this period the plasma and water remain practically immobile. Then intense expansion of the plasma into the cavity commences. With a radius $r_0 = 10$ cm the outer boundary of the plasma reaches the boundary of the water layer in a period of ~6 μsec. Almost all the plasma internal energy is transformed into kinetic energy, and the pressure within the plasma decreases by several orders of magnitude from the original value. This moment corresponds to curve 1 of Fig. 3, which also shows the pressure distribution in the medium at certain subsequent times. It should be noted that over such a brief time the parameters of the water are practically unchanged. In the following time because of propagation of compression and rarefaction waves in the gas and water the parameters of both media change intensely as can easily be seen in the same figure (curves 2-7). In particular, zones of reduced pressure are formed periodically both within the depths of the water (curves 2-5) and near the rigid spherical shell (curves 6, 7). It is interesting that the boundary between gas and liquid oscillates intensely. Its displacement with time for $r_0 = 1$ cm is shown in Fig. 4. The pressure on the wall also oscillates intensely, as shown by the solid curve of Fig. 5 (the dashed curve is for absence of radiation from the plasma). Formation of a reduced pressure zone which is preserved for a relatively lengthy period can easily be seen. For comparison Fig. 6 presents the time variation of pressure momentum at the wall with consideration of (solid curve) and in absence of (dashed curve) radiation.

Thus, direct calculation indicates the development of a unique oscillatory process in the retardant (water) and gas cavity. Consideration of viscosity and various losses upon reflection from the external boundary will make this process a self-damping one.

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